

# Катедра за Техничка Механика и Јакост на Материјалите

Факултет: Градежен

Предмет: **ЈАКОСТ НА МАТЕРИЈАЛИТЕ**

## МОМЕНТИ НА ИНЕРЦИЈА

Геометрички карактеристики на напречниот пресек

### 1. Тежиште на пресекот

$$x_c = \frac{\sum_{i=1}^3 A_i \cdot x_i}{\sum_{i=1}^3 A_i}$$

$$y_c = \frac{\sum_{i=1}^3 A_i \cdot y_i}{\sum_{i=1}^3 A_i}$$

$$A_1 = 2d \cdot 5d = 10d^2$$

$$C_1 (1d ; 2.5d)$$

$$A_2 = \frac{r^2 \cdot \pi}{4} = \frac{(2d)^2 \cdot \pi}{4} = 3.14d^2$$

$$x_2 = 2d - \frac{4r}{3\pi} = 2d - \frac{4 \cdot 2d}{3\pi} = 1.151d$$

$$y_2 = 5d - \frac{4r}{3\pi} = 5d - \frac{4 \cdot 2d}{3\pi} = 4.151d$$

$$C_2 (1.151d ; 4.151d)$$

$$A_3 = \frac{4d \cdot 3d}{2} = 6d^2$$

$$x_3 = 2d + \frac{4d}{3} = 3.333d$$

$$y_3 = \frac{3d}{3} = d$$

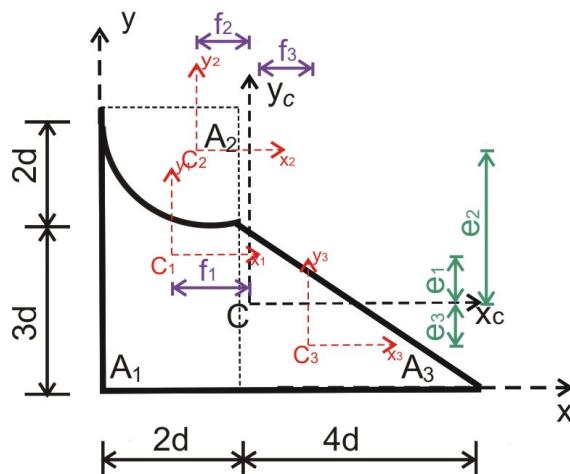
$$C_3 (3.333d ; d)$$

$$\sum A_i = A_1 - A_2 + A_3 = 10d^2 - 3.14d^2 + 6d^2 = 12.86d^2$$

$$x_c = \frac{\sum_{i=1}^3 A_i \cdot x_i}{\sum_{i=1}^3 A_i} = \frac{A_1 \cdot x_1 - A_2 \cdot x_2 + A_3 \cdot x_3}{A_1 - A_2 + A_3} = \frac{10d^2 \cdot 1d - 3.14d^2 \cdot 1.151d + 6d^2 \cdot 3.333d}{10d^2 - 3.14d^2 + 6d^2} = 2.052d$$

$$y_c = \frac{\sum_{i=1}^3 A_i \cdot y_i}{\sum_{i=1}^3 A_i} = \frac{A_1 \cdot y_1 - A_2 \cdot y_2 + A_3 \cdot y_3}{A_1 - A_2 + A_3} = \frac{10d^2 \cdot 2.5d - 3.14d^2 \cdot 4.151d + 6d^2 \cdot d}{10d^2 - 3.14d^2 + 6d^2} = 1.397d$$

$$C (2.052d ; 1.397d)$$



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## 2. Централни моменти на инерција

### 2.1 Аксијални моменти на инерција

$$Jx_c = Jx_1 - Jx_2 + Jx_3$$

$$Jx_1 = Jx_{10} + A_1 \cdot e_1^2$$

$$e_1 = y_1 - y_c = 2.5d - 1.397d = 1.103d$$

$$Jx_1 = \frac{2d \cdot (5d)^3}{12} + 10d^2 \cdot (1.103d)^2 = 20.833d^4 + 12.166d^4 = 33d^4$$

$$Jx_2 = Jx_{20} + A_2 \cdot e_2^2$$

$$e_2 = y_2 - y_c = 4.151d - 1.397d = 2.754d$$

$$Jx_2 = 0.055(2d)^4 + 3.14d^2 \cdot (2.754d)^2 = 0.88d^4 + 23.815d^4 = 24.695d^4$$

$$Jx_3 = Jx_{30} + A_3 \cdot e_3^2$$

$$e_3 = y_3 - y_c = d - 1.397d = -0.397d$$

$$Jx_3 = \frac{4d \cdot (3d)^3}{36} + 6d^2 \cdot (-0.397d)^2 = 3d^4 + 0.946d^4 = 3.946d^4$$

$$Jx_c = 33d^4 - 24.695d^4 + 3.946d^4 = 12.251d^4$$

$$Jy_c = Jy_1 - Jy_2 + Jy_3$$

$$Jy_1 = Jy_{10} + A_1 \cdot f_1^2$$

$$f_1 = x_1 - x_c = 1d - 2.052d = -1.052d$$

$$Jy_1 = \frac{5d \cdot (2d)^3}{12} + 10d^2 \cdot (-1.052d)^2 = 3.333d^4 + 11.067d^4 = 14.4d^4$$

$$Jy_2 = Jy_{20} + A_2 \cdot f_2^2$$

$$f_2 = x_2 - x_c = 1.151d - 2.052d = -0.901d$$

$$Jy_2 = 0.055(2d)^4 + 3.14d^2 \cdot (-0.901)^2 = 0.88d^4 + 2.549d^4 = 3.43d^4$$

$$Jy_3 = Jy_{30} + A_3 \cdot f_3^2$$

$$f_3 = x_3 - x_c = 3.333d - 2.052d = 1.281d$$

$$Jy_3 = \frac{3d \cdot (4d)^3}{36} + 6d^2 \cdot (1.281d)^2 = 5.333d^4 + 9.846d^4 = 15.179d^4$$

$$Jy_c = 14.4d^4 - 3.43d^4 + 15.179d^4 = 26.15d^4$$

$$Jy_c > Jx_c \Rightarrow \varphi \rightarrow \eta$$

## 2.2 Центрифугални моменти на инерција

$$Jx_c y_c = Jx_1 y_1 - Jx_2 y_2 + Jx_3 y_3$$

$$\begin{aligned} Jx_1 y_1 &= Jx_{10} y_{10} + A_1 \cdot e_1 \cdot f_1 = \\ &= 0 + 10d^2 \cdot 1.103d \cdot (-1.052d) = -11.604d^4 \end{aligned}$$

$$\begin{aligned} Jx_2 y_2 &= Jx_{20} y_{20} + A_2 \cdot e_2 \cdot f_2 = \\ &= -0.0165(2d)^4 + 3.14d^2 \cdot 2.754d \cdot (-0.901d) = -8.055d^4 \end{aligned}$$

$$\begin{aligned} Jx_3 y_3 &= Jx_{30} y_{30} + A_3 \cdot e_3 \cdot f_3 = \\ &= -\frac{(4d)^2 \cdot (3d)^2}{72} + 6d^2 \cdot (-0.397d) \cdot 1.281d = -5.051d^4 \end{aligned}$$

$$Jx_c y_c = -11.604d^4 + 8.055d^4 - 5.051d^4$$

$$\boxed{Jx_c y_c = -8.6d^4}$$

## 2.3 Главни централни оски на инерција

$$\tan 2\varphi = -\frac{2 \cdot Jx_c y_c}{Jx_c - Jy_c} = -\frac{2 \cdot (-8.6d^4)}{12.251d^4 - 26.15d^4} = -1.2375$$

$$2\varphi = -51.059^\circ$$

$$\boxed{\Rightarrow \varphi = -25.529^\circ}$$

## 2.4 Главни централни моменти на инерција

$$J_{1,2} = \frac{Jx_c + Jy_c}{2} \pm \frac{1}{2} \sqrt{(Jx_c - Jy_c)^2 + 4(Jx_c y_c)^2}$$

$$J_{1,2} = \frac{12.251d^4 + 26.15d^4}{2} \pm \frac{1}{2} \sqrt{(12.251d^4 - 26.15d^4)^2 + 4 \cdot (-8.6d^4)^2}$$

$$J_{1,2} = 19.2d^4 \pm \frac{1}{2} \sqrt{489.022d^8}$$

$$J_{1,2} = 19.2d^4 \pm 11.057d^4$$

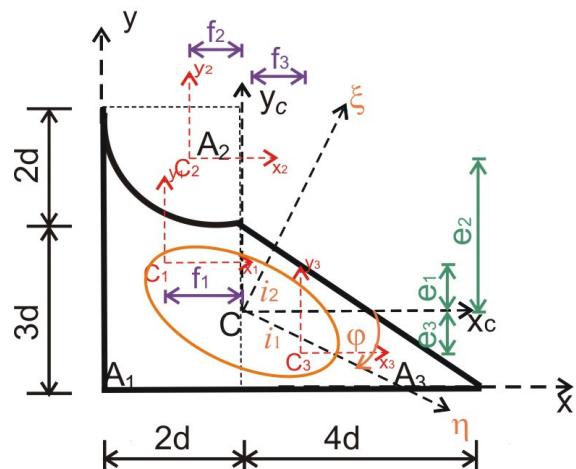
$$\boxed{J_1 = 30.257d^4}$$

$$\boxed{J_2 = 8.143d^4}$$

## 2.5 Елипса на инерција радиуси на елипсата:

$$i_1 = \sqrt{\frac{J_1}{A}} = \sqrt{\frac{30.257d^4}{12.86d^2}} = 1.534d$$

$$i_2 = \sqrt{\frac{J_2}{A}} = \sqrt{\frac{8.143d^4}{12.86d^2}} = 0.796d$$



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